Give Your Students the Gift of Mathematical Literacy

SUMMARY

Understanding mathematical concepts can be difficult for students who are accustomed only to listening to mathematics. But when you teach them to read, write and speak mathematics through the instructional strategies outlined here, their fluency in this new language will empower them.

This article addresses recommendations 1, 2, and 6 of the "Reading Next" and recommendations 1, 4, 8, and 9 of the "Writing Next" reports of the Alliance for Excellent Education and the Carnegie Corporation of New York. (See pages 95-96 and 98)

Did the students really get it?

As the class ended, I thought to myself that the lesson had gone really well. The students were all engaged, they had responded to my questions and asked some of their own. I just knew they all had understood the math concept! Then Dan approached and said, "Can I come for extra help after school today? I didn't understand a thing that you said!" As Dan left, the reality struck: If Dan, a "good" student, didn't get it, what about the others?

Unfortunately, this scenario takes place in far too many mathematics classrooms. As mathematics educators, we are fluent with our language, mathematics. We use it comfortably and do not feel that the words we speak or the symbols we use are at all foreign. We are mathematically literate because we can read, write, speak, and listen to mathematics with understanding. Yet, most students spend a disproportionate amount of time only listening to mathematics, so they do not get enough practice reading, writing, and speaking mathematics. How often do you think students participate in animated mathematical discussions at home with their parents over dinner or in the cafeteria with friends? How many read mathematics for pleasure or blog mathematically? The reality is that the only time most students are immersed in the language of mathematics is during math class.

Framework

The National Council of Teachers of Mathematics (NCTM), the New York State Standards, and the Partnership for 21st Century Skills hold communication — the ability to use language to express mathematical ideas precisely — as a vital skill for all students

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of mathematics. Since mathematics is so often conveyed in symbols, oral and written communication about mathematical ideas is not always recognized as an important part of mathematics education (NCTM Principles and Standards, 2000). Gay states, "Teachers need to be aware that their development of and use of vocabulary in the classroom contributes directly to students' understanding or misunderstanding of mathematical concepts" (2008, p. 221). To do a great job teaching the language of mathematics, teachers need to understand mathematical literacy as more than just vocabulary.

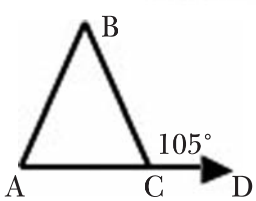
Let's begin with a formative assessment...

1. Simplify: $2 + 4 + 6 + 8 + 10$

2. Find the sum of the first five consecutive positive even integers.

3. Evaluate: $\sum_{x=1}^{5} 2x$

4. Find m$\angle$ABC if $\overline{AB} \cong \overline{BC}$.



Did you get 30 as a solution to all four? These problems model some of the multiple levels of symbolism within the language of mathematics. Rubenstein notes that "symbolism is a major dimension of the language of mathematics at all learning levels and is a tool for expressing relationships and for problem solving. Accessing and becoming fluent with symbolism is vital for mathematics success." *http://tsg.icme11.org/document/get/853*

Problems 1 and 3 characterize the common image of the symbolic nature of mathematics. The symbols in problem 1 are basic numerals and operations. Above the elementary level no decoding should be necessary. It's almost a "see and say" problem. Students do many worksheets designed at this level, never practicing higher level thinking or literacy skills. Problem 3 uses an advanced symbol which students many times can not even read let alone decode before understanding or manipulating its underlying concept. Problem 2 contains symbolism that can be characterized as "verbal." Again, students not only need to decode each word in the sentence, but must be able to synthesize all the words for mathematical

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comprehension of the problem. New York state students must be proficient at this level in order to be successful on mathematics state assessments. Finally, problem 4's symbolism is "visual." This time the student has to decode the diagram. The student should be able to verbalize the relationships shown before finding the calculated solution. This visual symbolism makes geometry challenging for most students. A single symbol or word may be enough to undermine a student's confidence, thus limiting that student's understanding of the mathematical problem.

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Figure 1:

Challenges of Mathematical Words and Symbols

SUM	3.05	COMMUTATIVE
$\frac{2}{3}$	8 - 3	=
Quotient	hypotenuse	π , i, or e

Figure 1 looks at some of the challenges of the words and symbols. (Adapted from Rubenstein, 2001, etc.)

As teachers we also need to use the correct mathematical language and model good literacy pedagogy. Sometimes we think we can make a concept easier for a student to understand if we substitute what we believe to be "easier" words or ideas for concepts. Usually the easier words move the student to a rote procedure instead of conceptual understanding.

Let us illustrate what we mean with examples. The commutative property of addition, 2 + 3 = 3 + 2, is introduced as early as the second grade. A teacher may think the word commutative "too hard" for a second grader and rename it as the "turn around" property. The next year, the third-grade teacher may call it the "switcheroo" property, and by fourth grade, when the teacher says "commutative," the student has no idea! Witherspoon states, "Only when children are exposed to an appropriate use of the word will they be able to interpret and use it correctly. If a mathematical symbol does not have the same meaning for everyone, it cannot be used as a communication tool" (1999, p. 397).

Let us examine another example. Here the challenge to students can be the fact that many words represent the same concept. In the mathematical statement 8 – 3, is the " – " read "take away," "minus," "subtract," or "negative"? Is the expression read as the difference of 8 and 3, the sum of 8 and negative 3, or from 8 subtract 3? At the middle level, both examples merge as teachers try to simplify the definition of subtraction of integers, "to subtract means to add the number's opposite," to "keep, change, change."

The simplification removes any operational context for the students.

The fraction $\frac{2}{3}$ is read by most teachers as "2 over 3," effectively negating the concept of division within a fraction. Students then have extreme difficulty with rewriting $\frac{2}{3}$ as either $2 \div 3$ or $3\overline{)2}$. Most students will define, and teachers accept, the definition of the denominator of a fraction as the "number on the bottom." That's *where* the denominator is, not *what* the denominator is.

The practice of speaking a mathematical symbol as if we are reading what we see is characterized in the number 3.05. Most teachers and students read what they see — "3-point-oh-5" instead of "3 and 5 hundredths". We then question why students have no understanding of the decimal place value.

In the case of the "=" sign, too many students understand "here comes the spot to put your answer, 3 + 2 = ___." As students advance, they experience confusion when asked, "What number completes the following statement: 7 + 5 + 3 = ____ + 5 + 7. Most students respond that the "____" would be filled in with 15! Was the concept of equality as balance ever understood by these students?

There is also π, i, and e. These are really not numbers, are they?

One of the major challenges can be the words of mathematics themselves. The word "sum" means total, yet it is a homophone to the word "some," which means less than all. "Quotient" is a math word that has no meaning outside of a math classroom. Math words like "hypotenuse" are hard to pronounce. Math vocabulary must be understood by users. Marzano and Pickering suggest some research-based strategies for effective vocabulary instruction that can be used in mathematics classes, such as: Effective vocabulary instruction does not rely on definitions, students must represent their knowledge of words in linguistic and nonlinguistic ways; effective vocabulary instruction involves the gradual shaping of word meaning through multiple exposures; students should discuss the terms they are learning; and students should play with words among others (Marzano, 2004; Marzano & Pickering, 2005).

Instructional Strategies

Practice Mathematical Language

Have students say the words out loud so they can hear how they sound and so they hear themselves pronounce the words. Words so specific to math need practice. Students need to go beyond hearing math. They need to read, write, and speak mathematics to be literate.

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If a mathematical symbol does not have the same meaning for everyone, it cannot be used as a communication tool.

Restructure the Math Class to Support Math Literacy

Rubenstein (Web reference, p. 1) believes that "teachers at all levels need to be conscious of the challenges that symbolization presents to students and have strategies for supporting students in gaining fluency." A heightened awareness to the complex nature of the language of mathematics, good English practice, and the use of correct and appropriate vocabulary to teach concepts is a good beginning, but other strategies are required. The class must be structured to allow the four domains of literacy an equal share of the time. The domains include Listening (L), Speaking (S), Reading (R) and Writing (W). Figure 2 compares the traditional stand-and-deliver classroom (left) to a literacy, vocabulary-conscious classroom in terms of the amount of time students spend in each domain.

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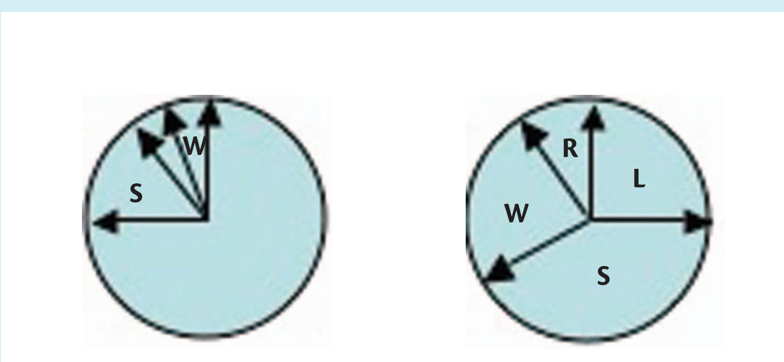
Figure 2:

"Traditional" vs. "Literacy Based"



Create a student-centered higher order thinking class

A better understanding of the multiple levels of mathematical representation causes lessons to be far more differentiated, better directed to students' multiple intelligences. In this type of class, concepts are constructed instead of presented. The classroom is student-centered and students are more often engaged in mathematical conversation, both with the teacher and other students, frequently using higher level language, questioning, and thinking skills. Teachers can become better at questioning. Murray suggests: "Developing the fine art of questioning takes planning, practice, reflection, and persistence. Coming up with questions that will push students to discover concepts and learn the related vocabulary demands even more of a teacher" (2004, pg. 41).

Develop language in the math class

Begin with simple changes, such as requiring proficient English skills in the math class. Students can be expected to answer questions in complete sentences. Those sentences should not begin with a pronoun, because it is harder for other students to follow a conversation when the noun has been left out. Instead of a student responding "180 degrees" when the teacher asks a question about the angle measures of a triangle, the student should be encouraged to respond, "Since the sum of the angle measures of a triangle is 180 degrees, then...." If students answer a question using non-math words, many teachers hear, understand, and accept those non-math words, but best literacy practice is to tell the student he or she has the right idea, but needs to use the proper mathematical vocabulary. Encourage students to look for the words on the classroom "word wall" or encourage others to help build the best mathematical answer. Literacy activities have to immerse students at all domains and make them take ownership of vocabulary. Just putting up a "word wall' will accomplish very little.

Restructure math worksheets

Restructure worksheets to include problems using vocabulary that students read, and then write mathematical sentences in response. Gay uses concept circles that "encourage students to study words critically, relating them conceptually to one another." (2008, p. 221) Figure 3 shows a concept circle that would work well as a class opener.

Students can respond with a variety of concepts: Even numbers of 2, 4, and 16 would not include 9 because it is odd; perfect squares of 4, 9, and 16 would not include 2; and 2, 4, and 9 as single-digit numbers would not include 16, a two-digit number. There is yet another concept that could categorize three of the four. Can you figure it out?

Murray provides many strategies to seamlessly promote classroom literacy, including a personal word wall in student notebooks, assigning students a chapter word on which they become the "expert." Students are responsible for that word's meaning through description, example and visual representation. They have to be able to use it in a complete and correct sentence, and maybe even use their body to do vocabulary "charades" ***I Have/ Who Has*** cards, ***Can of Words***, vocabulary as a ***Jeopardy!*** category during a chapter review, ***$250,000 Vocabulary***

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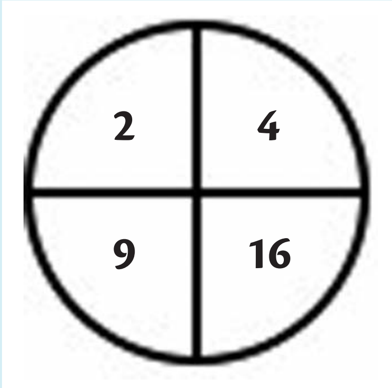
Figure 3:

Name a concept that relates three of the four numbers in the circle shown below. Explain why the fourth number does not belong.



One of the major challenges can be the words of mathematics themselves. The word "sum" means total, yet it is a homophone to the word "some," which means less than all.

Pyramid, Word Swat, and concept circles are just the beginning of classroom strategies that can be used.

Reflective Thoughts

Many mathematics classrooms are moving toward the literacy-based model. There are salient characteristics. Students are empowered by their fluency with the mathematical language, not giving up on problems because there are too many words or too many symbols. They read, write, and speak
mathematics with ease because that is what they have come to expect to do in math class each day. The teachers and students are a stronger community of learners because they use conversation to build mathematical concepts together.

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